TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 5 2022

Section 1 Calculator Free

Matrices

STUDENT'S NAME MARKING KEY [KRISZYK]

DATE: Tuesday 30th August **TIME:** 25 minutes **MARKS**: 22

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

Consider the matrices $\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ -1 & k \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} k & 1 \\ 0 & -2 \end{bmatrix}$ where $k \in \mathbb{R}$.

(a) Suppose $det(A) = 0$. Explain what this means. [1]

Matrix A is singular ✓

(b) Determine the value(s) of k given that:

(i) the matrix $\boldsymbol{AB}$ has no inverse. [3]

$AB = \begin{bmatrix} K & -3 \\ -K & -1-2K \end{bmatrix}$

$K(-1-2K) - (-3)(-K) = 0$ ✓

$-K - 2K^2 - 3K = 0$

$2K(K+2) = 0$

$K = 0$ ✓

$K = -2$

(ii) $\boldsymbol{B}^{-1} = \frac{1}{4}\boldsymbol{B}$ [3]

$\frac{1}{-2K}\begin{bmatrix} -2 & -1 \\ 0 & K \end{bmatrix} = \begin{bmatrix} \frac{K}{4} & \frac{1}{4} \\ 0 & -\frac{1}{2} \end{bmatrix}$ ✓

$\begin{bmatrix} \frac{1}{K} & \frac{1}{2K} \\ 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{K}{4} & \frac{1}{4} \\ 0 & -\frac{1}{2} \end{bmatrix}$ ✓

$\frac{1}{2K} = \frac{1}{4}$

$K = 2$ ✓

2. (4 marks)

Determine the transformed equation of the line $2y = 3x + 4$, when it is transformed by the matrix $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$.

$$y = \frac{3x}{2} + 2$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ \frac{3}{2}t + 2 \end{bmatrix} \quad ✓$$

$$= \begin{bmatrix} 6t + 4 \\ \frac{3}{2}t + 2 \end{bmatrix} \quad ✓$$

$$x' = 6t + 4$$

$$t = \frac{x-4}{6} \quad ✓$$

$$y' = \frac{3}{2}\left(\frac{x-4}{6}\right) + 2$$

$$y' = \frac{x}{4} + 1 \quad ✓$$

3. (4 marks)

Determine matrix **P** given that **AP** + **P** = **Q** + **BP** and **A** = $\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, **B** = $\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$, **Q** = $\begin{bmatrix} 1 & 0 \\ 3 & -6 \end{bmatrix}$.

$$AP + P - BP = Q$$

$$(A + I - B)P = Q \quad ✓$$

$$P = (A + I - B)^{-1} Q$$

$$= \begin{bmatrix} 1 & -4 \\ 0 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 3 & -6 \end{bmatrix} \quad ✓$$

$$= \frac{1}{3}\begin{bmatrix} 3 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & -6 \end{bmatrix} \quad ✓$$

$$P = \frac{1}{3}\begin{bmatrix} 15 & -24 \\ 3 & -6 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 5 & -8 \\ 1 & -2 \end{bmatrix} \quad ✓$$

4. (7 marks)

(a) If $\tan\theta = \frac{1}{2}$, determine the value of $\tan 2\theta$. [1]

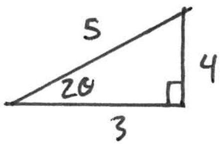
$$\tan 2\theta = \frac{2\tan\theta}{1-\tan\theta} = \frac{4}{3}$$

$$= \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}$$

(b) Determine the transformation matrix for a reflection in the line $y = \frac{x}{2}$. [2] 3

$$y = \frac{x}{2} \rightarrow y = \frac{1}{2}x \quad \text{i.e. } \tan\theta = \frac{1}{2}$$

$$\tan 2\theta = \frac{4}{3}$$



$$M = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \quad \text{or} \quad \frac{1}{5}\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$$

(c) When reflected in $y = \frac{x}{2}$, the image of the point $(a, -5)$ is $(2, b)$. Determine the value of a and the value of b. [3] 4

$$\frac{1}{5}\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} a \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ b \end{bmatrix}$$

$$\begin{bmatrix} 3a-20 \\ 4a+15 \end{bmatrix} = \begin{bmatrix} 10 \\ 5b \end{bmatrix}$$

$$a = 10$$

$$b = 11$$

TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 5 2022

Section 2 Calculator Assumed

Matrices

STUDENT'S NAME MARKING KEY [KRISZYK]

DATE: Tuesday 30th August **TIME:** 20 minutes **MARKS**: 21

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (10 marks)

Transformation $\boldsymbol{Q}$ is an anti-clockwise rotation about the origin of $\frac{4\pi}{3}$.

(a) Represent transformation $\boldsymbol{Q}$ as a 2×2 matrix using exact values. [2]

$$Q = \begin{bmatrix} \cos\left(\frac{4\pi}{3}\right) & -\sin\left(\frac{4\pi}{3}\right) \\ \sin\left(\frac{4\pi}{3}\right) & \cos\left(\frac{4\pi}{3}\right) \end{bmatrix} \checkmark = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} \checkmark$$

A second Transformation Matrix P is given by $\boldsymbol{P} = \begin{bmatrix} 2 & 0 \\ 0 & 0.25 \end{bmatrix}$

(b) Describe the transformation represented by the matrix $\boldsymbol{P}$. [2]

1. Dilation parallel to x-axis S.F = 2. ✓
2. Dilation parallel to y-axis S.F = $\frac{1}{4}$ ✓

(c) The triangle $A(1,2)$, $B(-1,4)$, $C(-6,8)$ is transformed by the matrix $\boldsymbol{Q}$ followed by the matrix $\boldsymbol{P}$ to the triangle $A''B''C''$. Find coordinates of A'', B'' and C'', correct to 2 decimal places. [2]

$$\begin{array}{ccc} P & Q & \\ \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{4} \end{bmatrix} & \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} & \begin{bmatrix} 1 & -1 & -6 \\ 2 & 4 & 8 \end{bmatrix} \end{array} \checkmark$$

A'' $(2.46, -0.47)$

B'' $(7.93, -0.28)$ ✓

C'' $(19.86, 0.30)$

(d) Determine the area of triangle $A''B''C''$ [2]

$\det PQ = 0.5$

Area $ABC = 1$

$\therefore$ Area $A''B''C'' = 0.5$

(e) Determine the single matrix M that will transform the triangle $A''B''C''$ back to the triangle ABC using exact values. [2]

$$PQ^{-1} = \begin{bmatrix} -1 & \sqrt{3} \\ \frac{\sqrt{3}}{8} & \frac{-1}{8} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{4} & -2\sqrt{3} \\ +\frac{\sqrt{3}}{4} & -2 \end{bmatrix}$$

6. (7 marks)

A system of equations is given by

$$4x + ay - 9 = 0$$
$$-2x + 3y + 3 = 0$$

(a) Let the constant $a = -5$.

(i) Express the system in matrix form $AX = B$.

[2]

$$\begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \end{bmatrix} \checkmark$$

$\checkmark$

(ii) Determine A^{-1} and demonstrate use of matrix algebra to solve the system for X.

[3]

$$A = \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ -3 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 6$ $\checkmark$

$y = 3$

(b) Determine the value of a for which the system has no solution and comment on the relationship between the two lines that form the system when a has this value. [2]

$$|A| = 0$$

$$\begin{bmatrix} 4 & a \\ -2 & 3 \end{bmatrix}$$

$\det A = 12 + 2a$

$0 = 12 + 2a$

$a = -6$ $\checkmark$

lines are parallel $\checkmark$

7. (4 marks)

Consider matrix $\boldsymbol{P} = \begin{bmatrix} p & 3p-1 \\ 1 & p \end{bmatrix}$. Determine the value(s) of p such that det $(\mathbf{P})$ = det $(\mathbf{P}^{-1})$

$\det(P) = p^2 - 3p + 1$ ✓

Via CP $\det(P^{-1}) = \dfrac{p^2 - 3p + 1}{(p^2 - 3p + 1)^2}$ ✓

$= \dfrac{1}{p^2 - 3p + 1}$

Via CP Solve $p^2 - 3p + 1 = \dfrac{1}{p^2 - 3p + 1}$ ✓

$p = 0, 1, 2, 3$ ✓